NBS

Technical Note No. 182

A NOTE ON ANTIPODAL FOCUSSING

JAMES R. WAIT

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to government agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. Research projects are also performed for other government agencies when the work relates to and supplements the basic program of the Bureau or when the Bureau's unique competence is required. The scope of activities is suggested by the listing of divisions and sections on the inside of the back cover.

Publications

The results of the Bureau's research are published either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau publishes three periodicals available from the Government Printing Office: The Journal of Research, published in four separate sections, presents complete scientific and technical papers; the Technical News Bulletin presents summary and preliminary reports on work in progress; and the Central Radio Propagation Laboratory Ionospheric Predictions provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: Monographs, Applied Mathematics Series, Handbooks, Miscellaneous Publications, and Technical Notes.

A complete listing of the Bureau's publications can be found in National Bureau of Standards Circular 460, Publications of the National Bureau of Standards, 1901 to June 1947 ($1.25), and the Supplement to National Bureau of Standards Circular 460, July 1947 to June 1957 ($1.50), and Miscellaneous Publication 240, July 1957 to June 1960 (includes Titles of Papers Published in Outside Journals 1950 to 1959) ($2.25); available from the Superintendent of Documents, Government Printing Office, Washington D.C. 20402.

NATIONAL BUREAU OF STANDARDS

Technical Note 182

Issued August 20, 1963

A NOTE ON ANTIPODAL FOCUSSING

James R. Wait

Central Radio Propagation Laboratory

NBS Boulder Laboratories

Boulder, Colorado

For sale by the Superintendent of Documents, U. S. Government Printing Office
Washington, D.C. 20402

Price 15¢

A NOTE ON ANTIPODAL FOCUSSING

James R. Wait

There has been some interest shown recently in the practical significance of the antipodal focussing in VLF propagation. If the earth and the ionosphere were perfectly concentric spherical surfaces, theory indicates that the amplitude of the field should build up to a maximum at the geographic antipodal point of the transmitter. In the language of optics, this point may be called an axial caustic.

Experimental evidence of some manner of focussing at the antipode was obtained by Round, Tremellen, Eckersley, and Lunnon [1925] in an early series of measurements. Their field strength data were obtained at frequencies in the range from 20 kc/s to 30 kc/s. More recently, Crombie [1958] and Bickel, et al, [1963] have found clear evidence of antipodal focussing. As predicted by theory [e. g., Wait, 1962], the field strength E was found to vary approximately as the inverse square root of the distance from the antipode. There was also some indication that a standing wave pattern existed in the antipodal region. Unfortunately, this pattern did not always exhibit a consistent form and the spacing between the nulls departed considerably from the half-wavelength expected on the basis of simple theory. It has been indicated by Crombie [1963] and by Bickel [1963] that asymmetry in the earth-ionosphere cavity may impair the focussing to some extent. Generally, they used arguments based on geometrical optics.

It is the purpose of the present paper to discuss this matter further. To shed some light on the subject, a very simple perturbation of the ideal model is considered. Essentially, it is assumed that the illumination of the antipodal region is non-uniform.

The region around the geographic antipode is depicted in Fig. 1. For present purposes, the antipodal region may be regarded as a radial transmission line and thus local cylindrical coordinates (ρ, ϕ, z) are convenient. The earth's surface is $z = 0$, while the effective lower boundary of the ionosphere is h. For some concentric region $\rho \leqq \rho_0$, it is assumed that the ionosphere height h and its electric properties are constant. Furthermore, the surface impedance of the earth's surface at $z = 0$ is regarded as a constant. This assumption of local uniformity around the antipode greatly simplifies the subsequent discussion.

An appropriate form of solution for the vertical component of the electric field, for a waveguide mode of order n, is given by

$$E^{(n)}(\rho, \phi, z) = \sum_{m=-\infty}^{+\infty} A_m J_m(k\rho S_n) e^{im\phi} f_{m,n}(z) , \tag{1}$$

where J_m is the Bessel function of the first kind of order m and argument $k\rho S_n$ in which kS_n is the propagation constant for a waveguide mode of order n. The function $f_{m,n}(z)$ is a height-gain function for mode n and its specific form is of no concern at the moment; however, in general, it depends on both m and n. If the field in the antipodal region were perfectly symmetrical, $\partial/\partial\phi = 0$ and then

$$E^{(n)}(\rho, \phi, z) = A_o J_o(k\rho S_n) f_{o,n}(z). \tag{2}$$

This particular situation has been discussed extensively in the literature [e.g., Wait, 1958, 1962; Norton, 1959]. In most applications to VLF propagation to great ranges, it is permissible to consider only one mode and thus the affix n may be dropped in what follows.

Equation (2) has a clear physical significance when J_o is replaced by the first term of its asymptotic expansion. Thus for $|k\rho S| >> 1$,

$$J_o(k\rho S) \sim \sqrt{\frac{2}{\pi k\rho S}} \; \frac{e^{ik\rho S} e^{-i\pi/4} + e^{-ik\rho S} e^{i\pi/4}}{2} , \tag{3}$$

which can be regarded as the superposition of two travelling waves.

At the radial distance $\rho = \rho_o$, the field has the form

$$E(\rho_o, \phi, z) = \sum_{m=-\infty}^{+\infty} A_m J_m(k\rho_o S) e^{im\phi} f_m(z) , \tag{4}$$

where the notation has been simplified in accordance with the discussion above. On physical grounds we will now assume something about the variation of $E(\rho_o, \phi, z)$. Then, the resulting form of $E(\rho, \phi, z)$, when $\rho < \rho_o$, will be discussed.

By multiplying both sides of equation (4) by $e^{-im'\phi}$ and integrating with respect to ϕ, over the range 0 to 2π, one is lead readily to the formula

$$A_m = \frac{1}{2\pi J_m(k\rho_o S)} \int_0^{2\pi} E(\rho_o, \phi, z) e^{-im\phi} d\phi , \tag{5}$$

if one notes that

$$\int_0^{2\pi} e^{i(m-m')\phi}\, d\phi = \begin{cases} 2\pi & \text{if } m = m' \\ 0 & \text{if } m \neq m' \end{cases} \tag{6}$$

Thus, formally, the coefficient A_m may be found from the specific form assumed for the electric field around the ring $\rho = \rho_o$.

As a simple example, which leads to an embarrassingly simple formula, it will be assumed that the illumination on the ring (at $\rho = \rho_o$) is constant except for a finite sector. Such a situation could be envisaged by imagining that the energy, propagated from the transmitter, passed, in part, over a very lossy region of the earth. Ignoring any diffraction effects, it will then be supposed that the great circle paths in the angular region $\phi_1 < \phi < \phi_2$ are paths of complete absorption. Thus, over this interval on the ring $\rho = \rho_o$,

$$E(\rho_o, \phi, z) \cong \left(\frac{1}{\pi k \rho S}\right)^{\frac{1}{2}} e^{-i(k\rho S_n - \pi/4)} A_o \ , \tag{7}$$

which is the contribution from the long great circle paths which are propagating from the direction $\pi + \phi_1 > \phi > \pi + \phi_2$ away from the antipode.

Using equation (5), it now follows that

$$\begin{aligned} A_m = \frac{1}{2\pi J_m(k\rho_o S)} \Bigg\{ & \int_0^{2\pi} J_o(k\rho_o S)\, e^{-im\phi}\, d\phi \\ & + \int_{\phi_1}^{\phi_2} \left[E^+(\rho_o) - J_o(k\rho_o S)\right] e^{-im\phi}\, d\phi \\ & + \int_{\pi+\phi_1}^{\pi+\phi_2} \left[E^-(\rho_o) - J_o(k\rho_o S)\right] e^{-im\phi}\, d\phi \Bigg\} \end{aligned} \tag{8}$$

where

$$E^{\pm}(\rho_o) - J_o(k\rho_o S) \cong -\left(\frac{1}{2\pi k \rho_o S}\right)^{\frac{1}{2}} e^{\pm i(k\rho_o S - \pi/4)} \tag{9}$$

The integrations can be carried out readily to yield, for $m \neq 0$,

$$A_m = -\frac{1}{2\pi J_m(k\rho_o S)} \frac{1}{(2\pi k \rho_o S)^{\frac{1}{2}}} \left\{ e^{i(k\rho_o S - \pi/4)} e^{-im\phi_o} \right.$$

$$\left. + e^{-i(k\rho_o S - \pi/4)} e^{im\pi} e^{-im\phi_o} \right\} \Delta\phi \Lambda_m , \tag{10}$$

where

$$\phi_o = (\phi_1 + \phi_2)/2 ,$$

$$\Delta_\phi = \phi_2 - \phi_1 ,$$

and

$$\Lambda_m = \frac{\sin(m\Delta\phi/2)}{(m\Delta\phi/2)} .$$

In view of the already imposed condition that $|k\rho_o S| >> 1$, the Bessel function J_m in the denominator may be replaced by its asymptotic expansion. Thus, for $m \neq 0$,

$$A_m \cong -\frac{e^{im\pi/2}}{2\pi} e^{-im\phi_o} \Delta\phi \Lambda_m . \tag{11}$$

Within the same approximation, it readily follows that

$$A_o \cong 1 - \frac{\Delta\phi}{2\pi} . \tag{12}$$

Employing the above explicit formulas for the coefficient A_m, the resulting behavior of the field in the vicinity of the antipode may be deduced from equation (4). To illustrate the nature of the problem, it is assumed that Λ_m can be replaced by unity. This would be well justified if Δ is small compared with unity and if $k\rho S$ is not too large. At the same time, the height-gain function $f_m(z)$ is replaced by unity. With these simplifications,

$$E(\rho, \phi) \cong J_o(k\rho S) - q e^{ik\rho S \cos(\phi - \phi_o)} , \tag{13}$$

where $q = \Delta\phi/2\pi$. It is quite clear that the total field is the superposition of the ideal symmetric antipodal field and a plane wave, of relative amplitude q, incident from the direction $\phi = \phi_o$. The negative sign preceding q is to indicate that the plane wave field is to be subtracted from the ideal or undisturbed antipodal field.

In a certain sense, the plane wave term can be regarded as the most basic type of perturbation. In fact, a generally perturbed field could be written in the form

$$E(\rho, \phi) \cong J_o(k\rho S) - \sum_s q_s e^{ik\rho S \cos(\phi_o - \phi_s)} , \tag{14}$$

where the summation over planes incident at various angles $\phi = \phi_s$. Furthermore, the coefficient q_s may be complex to account for relative phase shifts between the perturbing plane waves and the ideal antipodal field.

In order to characterize the amplitude behavior of the resultant field for a single disturbing plane wave, the function

$$E(x) = J_o(x) - q\, e^{ix \cos \phi} , \tag{15}$$

is considered, where $x = k\rho S = 2\pi\rho/\lambda_e$ in terms of the effective wavelength λ_e. While strictly speaking, S has a small imaginary part [Wait, 1962], it may be regarded as real for the present discussion.

The magnitude of $E(x)$, expressed in db, is plotted in Fig. 2 for $q = 0.1$ for a range of azimuth angles ϕ. Because of certain symmetry properties, the curves are identical at ϕ and $180^\circ - \phi$. Also, although not shown explicitly on the curves, symmetry exists about the direction of $\phi = 0^\circ$. Thus, the results in Fig. 2 are also applicable for negative angles.

The curves, in Fig. 2, for $\phi = 0^\circ$ (and 180°) are almost identical to the ideal antipodal field pattern characterized by the Bessel function $J_o(2\pi\rho/\lambda_e)$ alone. Here, the variation along the radial is in the direction of propagation of the perturbing wave. At other angles, it is apparent that the pattern becomes distorted and the spacing between nulls is modified to some extent.

The effect of varying q, the amplitude of the perturbing wave is shown in Fig. 3. For these curves, ϕ is fixed at 45°. It is evident that for the larger q values, the deep nulls disappear. In this situation the symmetric J_o antipodal field can be regarded as a perturbation to the plane wave field.

The graphical results shown in Figs. 2 and 3 are intended to indicate merely that the field pattern near the antipodal may become extremely complicated. Many other intriguing patterns can be generated by using other combinations of the parameters. Also, equally interesting, are the resulting phase variations near the antipode which may be calculated from equation (15).

I would like to thank Mrs. C. M. Jackson for carrying out the calculations shown graphically in Figs. 2 and 3.

References

Bickel, J. E. et al.(1963). Personal communication.

Crombie, D. D. (1958). Differences between east-west and west-east propagation on VLF signals over long distances, J. Atmos. and Terrest. Phys. 12, 110-117.

Crombie, D. D. (1963). Personal communications.

Norton, K. A. (June 1959). Transmission loss in radio propagation, Part II, NBS Technical Note No. 12.

Round, H. J. T., T. L. Eckersley, K. Tremellen, and F. C. Lunnon (1925). Report on measurements made on signal strength at great distances during 1922 and 1923 by an expedition sent to Australia, J. Instn. Elect. Engrs. 63, 933-1011.

Wait, J. R. (1958, 1960). Diffractive corrections to the geometrical optics of low frequency propagation, Proc. Liege Conference on Wave Propagation (1958); also published in Electromagnetic Wave Propagation, edited by M. Desirant and J. L. Michiels (Academic Press, 1960).

Wait, J. R. (1962). Electromagnetic waves in stratified media, Chapters VI and VIII (Pergamon Press, London, New York).

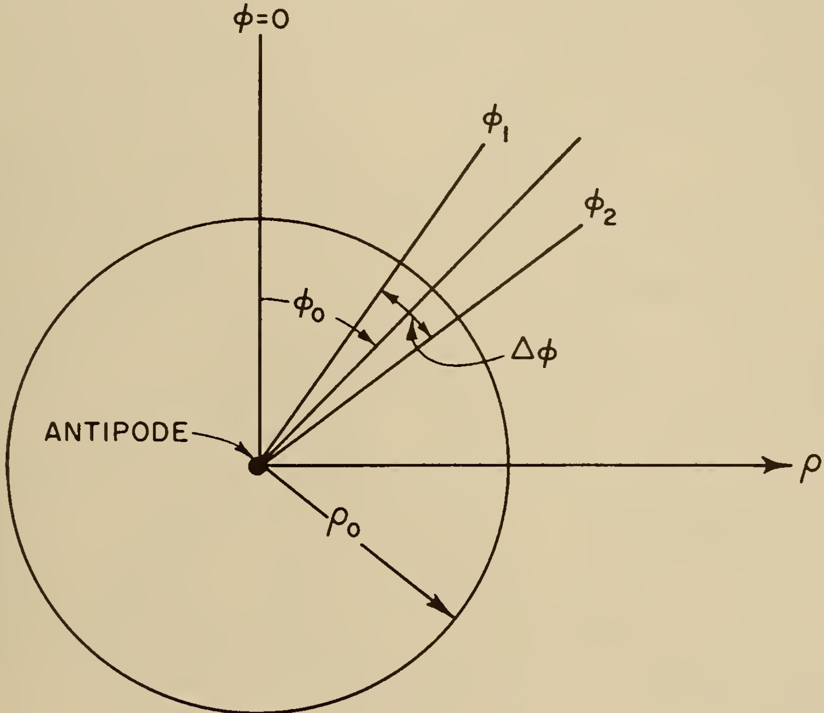

Fig. 1 Coordinate system for antipodal region.

RESULTANT ANTIPODAL FIELD FOR $q = 0.1$ AND $b = 0$

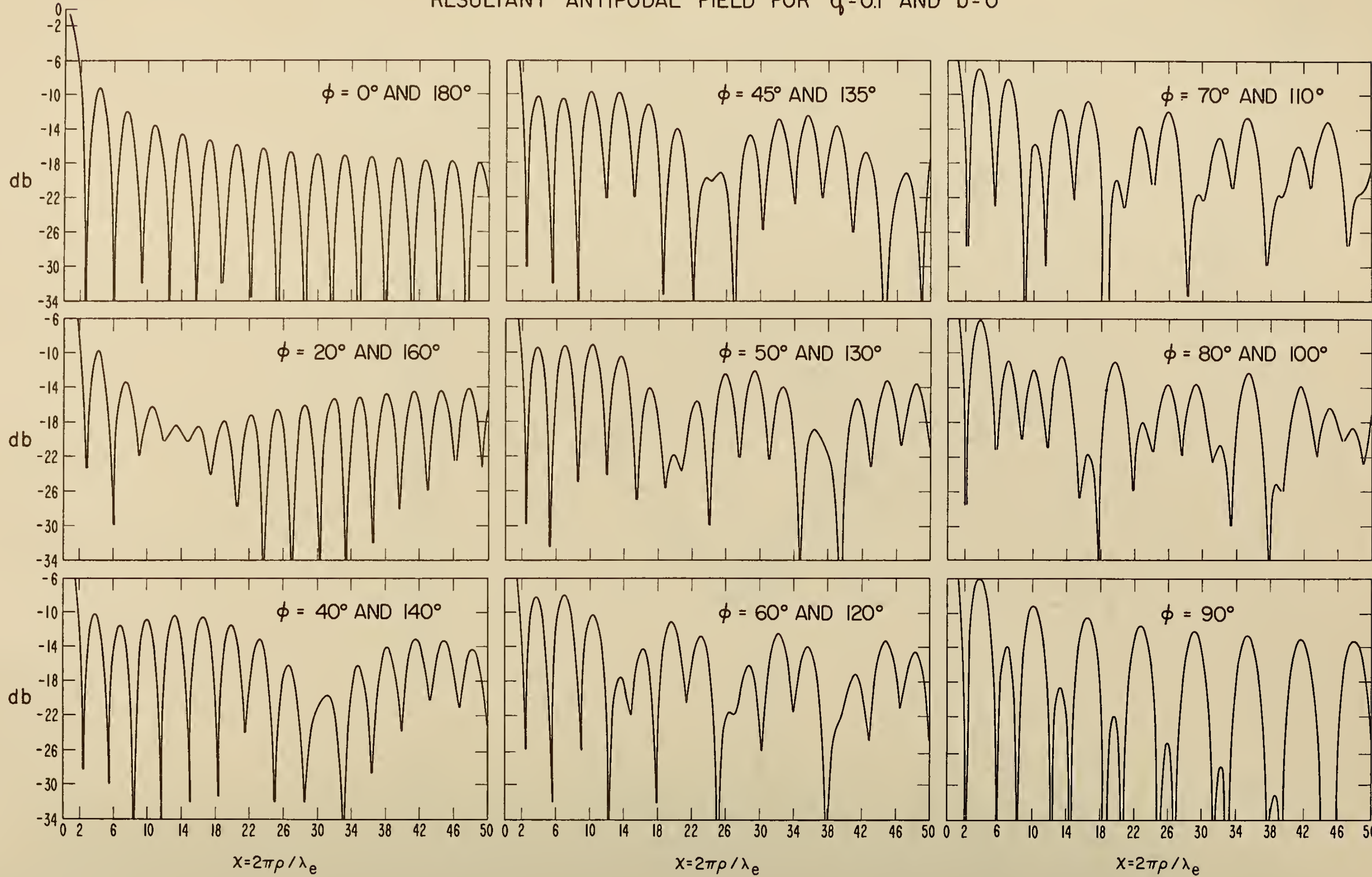

Fig. 2

RESULTANT ANTIPODAL FIELD FOR $\phi=45°$

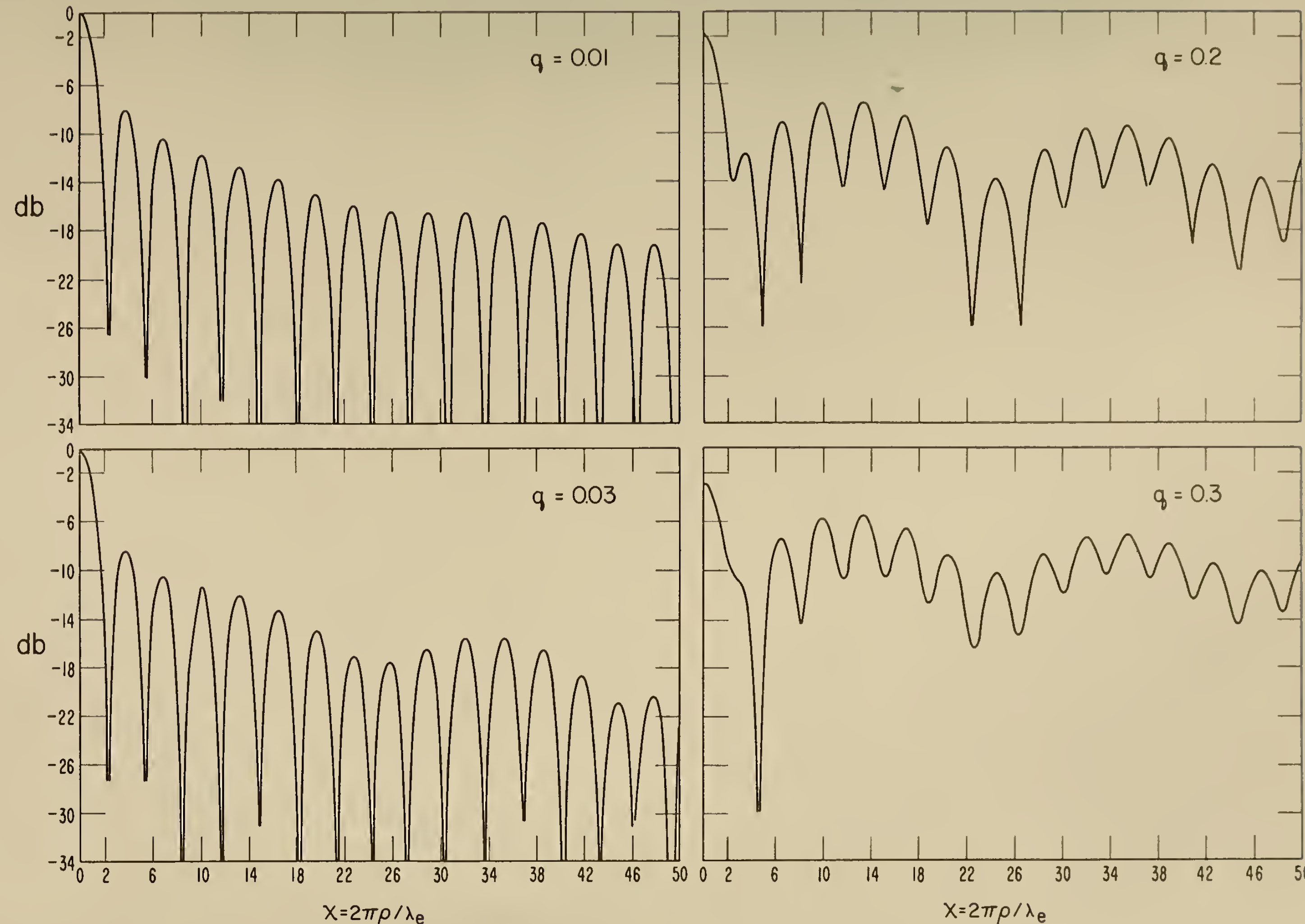

Fig. 3

U. S. DEPARTMENT OF COMMERCE
Luther H. Hodges, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D. C.

Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics. High Voltage.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

Heat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics.

Radiation Physics. X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research. Crystal Chemistry.

Mechanics. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

Polymers. Macromolecules: Synthesis and Structure. Polymer Chemistry. Polymer Physics. Polymer Characterization. Polymer Evaluation and Testing. Applied Polymer Standards and Research. Dental Research.

Metallurgy. Engineering Metallurgy. Microscopy and Diffraction. Metal Reactions. Metal Physics. Electrolysis and Metal Deposition.

Inorganic Solids. Engineering Ceramics. Glass. Solid State Chemistry. Crystal Growth. Physical Properties. Crystallography.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials. Metallic Building Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems. Components and Techniques. Computer Technology. Measurements Automation. Engineering Applications. Systems Analysis.

Atomic Physics. Spectroscopy. Infrared Spectroscopy. Far Ultraviolet Physics. Solid State Physics. Electron Physics. Atomic Physics. Plasma Spectroscopy.

Instrumentation. Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Physical Chemistry. Thermochemistry. Surface Chemistry. Organic Chemistry. Molecular Spectroscopy. Elementary Processes. Mass Spectrometry. Photochemistry and Radiation Chemistry.

Office of Weights and Measures.

BOULDER, COLO.

Cryogenic Engineering Laboratory. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Cryogenic Technical Services.

CENTRAL RADIO PROPAGATION LABORATORY

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Frequency Utilization. Modulation Research. Antenna Research. Radiodetermination.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. High Latitude Ionosphere Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.

RADIO STANDARDS LABORATORY

Radio Physics. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time-Interval Standards. Radio Plasma. Millimeter-Wave Research.

Circuit Standards. High Frequency Electrical Standards. High Frequency Calibration Services. High Frequency Impedance Standards. Microwave Calibration Services. Microwave Circuit Standards. Low Frequency Calibration Services.

NBS